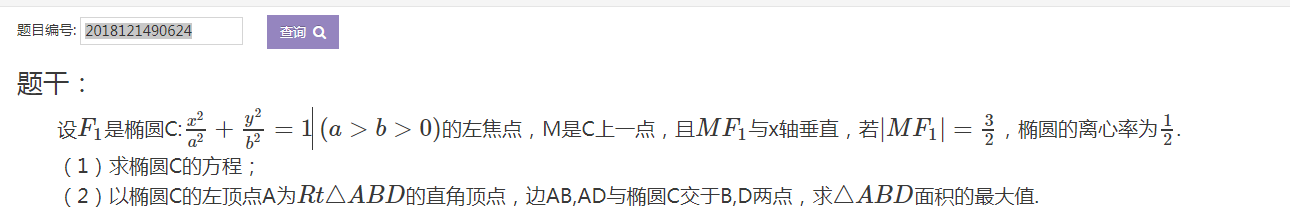
**Example 1: 2018121490624 A math problem for analytic geometry** 

Let F\_1 be the left focus of ellipse C:x^2/a^2+y^2/b^2=1(a>b>0), Point M lies on C, and MF\_1 is perpendicular to the X-axis, if |MF\_1|=3/2, the eccentricity of ellipse C is 1/2.

(1) Find the standard equation of ellipse C.

(2) If the left vertex of ellipse C is point A, and the right vertex of △ABD is point A, AB intersects ellipse C at B, and AD intersects ellipse C at D. Find the maximum value of area of △ABD.

Human-like solving processes:

Question (1):

(1)∵ line F\_1M⊥line X:y=0

(2)∴ by(1): analytic of function F\_1M is x=x\_F\_1M

(3)∴ by(1): analytic of function X is y=0

(4)∴ by(1,2,3): x\_F\_1M=0

(5)∵ the focus of C is F\_1.

(6)∵ analytic of ellipse C is ((x^2)/(a^2))+((y^2)/(b^2))=1

(7)∴ by(5): point F\_1

(8)∴ by(5,6,7): point F\_1(-(a^2-b^2)^(1/2),0)

(9)∴ by(3,8): point F\_1 (-(a^2-b^2)^(1/2), 0) is on line X: y = 0

(10)∴ by(2): point F\_1 is on line F\_1M: x = x\_F\_1M

(11)∴ by(9,10): line X:y=0 and line F\_1M:x=x\_F\_1M crossing at point F\_1(-(a^2-b^2)^(1/2),0)

(12)∴ by(11): point F\_1(-(a^2-b^2)^(1/2), 0) is on line F\_1M:x=x\_F\_1M

(13)∴ by(4,8,12): analytic of function F\_1M is x=-(a^2-b^2)^(1/2)

(14)∴ by(13): point M is on line F\_1M:x=-(a^2-b^2)^(1/2)

(15)∵ point M is on ellipse C

(16)∴ by(15): point M

(17)∴ by(15,16): point M(s\_M, t\_M)

(18)∴ by(13,14,17): s\_M+(a^2-b^2)^(1/2)=0

(19)∴ by(6): a>b

(20)∴ by(6,15,17): s\_M≥-a

(21)∴ by(6,15,17): t\_M≥-b

(22)∵ F\_1M=(3/2)

(23)∴ by(22): |vector F\_1M| is (3/2)

(24)∴ by(23): vector F\_1M is (s\_M+(a^2-b^2)^(1/2), t\_M)

(25)∴ by(24): vector F\_1M

(26)∴ by(8,17,25): vector F\_1M=(s\_M+(a^2-b^2)^(1/2), t\_M)

(27)∴ by(23,26): (a^2-b^2+2\*s\_M\*(a^2-b^2)^(1/2)+s\_M^2+t\_M^2)^(1/2)=3/2

(28)∴ by(6): a^2≠b^2

(29)∴ by(5,6): Point F\_1 is on line X:y=0

(30)∴ by(5,29): The focus of C is on X axis.

(31)∴ by(6,30): a^2>b^2

(32)∴ by(6): a>0

(33)∴ by(6,15,17): t\_M≤b

(34)∴ by(6,15,17): s\_M^2/a^2+t\_M^2/b^2-1=0

(35)∴ by(6): b>0

(36)∴ by(6): focal length of conic C is 2\*C\_3

(37)∴ by(6,36): C\_3>0

(38)∴ by(4,14,17): analytic of function F\_1M is x=s\_M

(39)∴ by(38): point F\_1 is on line F\_1M:x=s\_M

(40)∴ by(8,38,39): -(a^2-b^2)^(1/2)-s\_M=0

(41)∴ by(6,15,17): s\_M≤a

(42)∴ by(18,19,20,21,27,28,31,32,33,34,35,37,40,41): s\_M=-1,a=2,b=3^(1/2),t\_M=3/2 or s\_M=-1,a=2,b=3^(1/2),t\_M=-3/2

Discussions in different conditions:

Condition 1

when [${s}\_{M}=(-1)$, $a=2$, $b=\sqrt{3}$, ${t}\_{M}=\frac{3}{2}$]:

(1)∵ b=3^(1/2)

(2)∵ a=2

(3)∵analytic of ellipse C is ((x^2)/(a^2))+((y^2)/(b^2))=1

(4)∴ by(2,3): analytic of ellipse C is 1/4\*(b^2\*x^2+4\*y^2)/b^2=1

(5)∴ by(1,4): analytic of ellipse C is 1/4\*x^2+1/3\*y^2=1

when [${s}\_{M}=(-1)$, $a=2$, $b=\sqrt{3}$, ${t}\_{M}=(-\frac{3}{2})$]:

Condition 2

The same as Condition 1

To sum up, [the standard equation of ellipse C is x^2/4+y^2/3=1]

Question (2):

(1)∵ the standard equation of ellipse C is x^2/4+y^2/3=1

(2)∵ the left vertex of ellipse C is point A

(3)∴ by (1,2): A(-2,0)

(4)∵ line AB intersects ellipse C at B

(5)∴ by (3,4): the equation of  function AB is y=k\_AB\*(x+2)

(6)∵ line AD intersects ellipse C at D

(7)∴ by (3,6): the equation of  function AD is y=k\_AD\*(x+2)

(8)∵ let B(x\_B, y\_B)

(9)∴ by (1,4,8): x\_B^2/4+y\_B^2/3=1

(10)∴ by (3,5,8): k\_AB=(y\_B-0)/(x\_B+2)

(11)∵ let D(x\_D, y\_D)

(12)∴ by (1,6,11): x\_D^2/4+y\_D^2/3=1

(13)∴ by (3,7,11): k\_AD=(y\_D-0)/(x\_D+2)

(14)∵ segment AB

(15)∴ by (3,8,14): AB=((x\_B+2)^2+(y\_B-0)^2)^(1/2)

(16)∵ segment AD

(17)∴ by (3,11,16): AD=((x\_D+2)^2+(y\_D-0)^2)^(1/2)

(18)∵ Rt△ABD(vertex is point A)

(19)∴ by (18): Rt∠BAD

(20)∴ by (19): AD⊥AB, foot point is A

(21)∴ by (20): segment AB is the height of △ABD

(22)∴ by (21): S\_△ABD=((1/2)\*AD)\*AB

(23)∵ S\_△ABD=v\_0

(24)∴ by (15,17,22,23): v\_0=1/2\*((x\_D+2)^2+(y\_D-0)^2)^(1/2)\*((x\_B+2)^2+(y\_B-0)^2)^(1/2)

(25)∴ by (20): k\_AB\*k\_AD=-1

(26)∴ by (9,10,12,13,15,17,22,23,24,25): the maximum value of S\_△ABD is 144/49

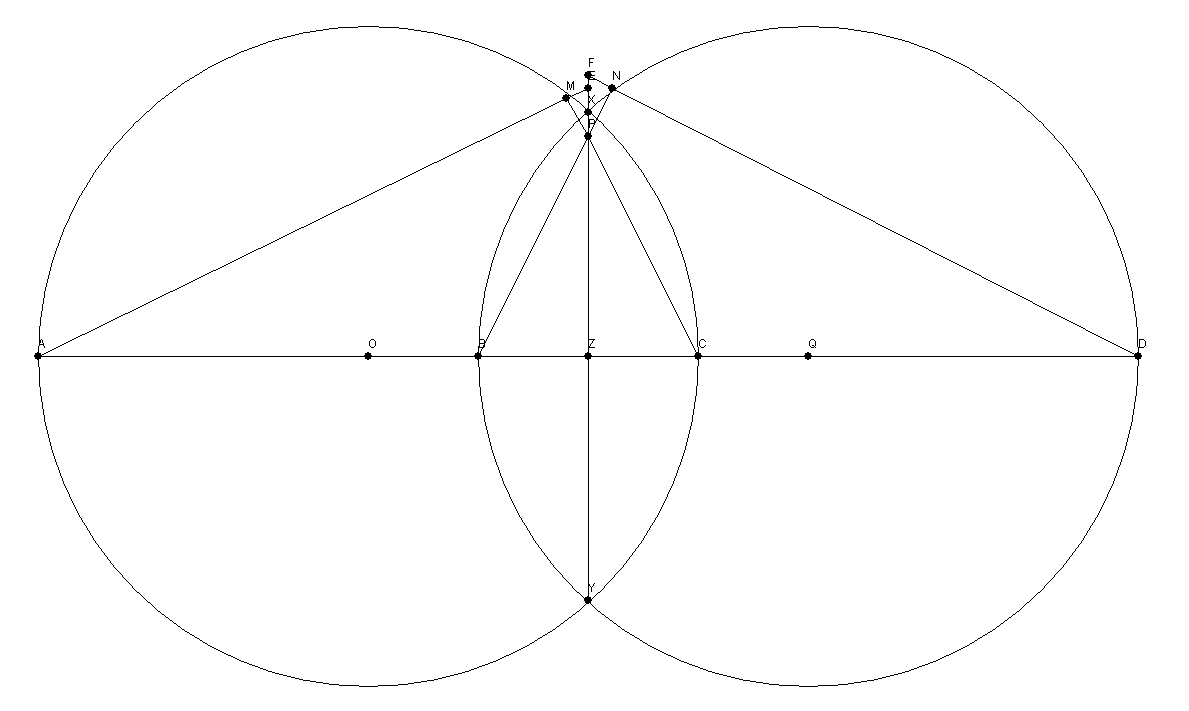
Example 1 shows the solving processes of different strategies.

**Example 2: An Olympic math problem for 2D geometry**

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1. Problem description:

Suppose point A, B, C and D are four different points arranged in turn on a straight line, the line intersects with the circle O having a diameter AC at point X, and intersects with the circle Q having a diameter BD at point Y. Line XY intersects BC with point Z, if point P is a point different from Z on line XY, the line CP intersects with the circle O having a diameter AC at point C and M, the line BP intersects with the circle Q having a diameter BD at point B and N. Prove : AM, XY and DN three lines intersect at one point.



1. Graphic information:

{"substems":[],"stem":{"pictures":[{"variable-equals":{},"picturename":"","circles":[{"center":"O","pointincircle":"A###M###X###C###Y"},{"center":"Q","pointincircle":"B###X###N###D###Y"}],"collineations":{"0":"A###O###B###Z###C###Q###D","1":"F###E###X###P###Z###Y","2":"M###P###C","3":"N###P###B","4":"A###M###E","5":"F###N###D"},"coordinates":{"A":"0.00,0.00","B":"40.00,0.00","C":"60.00,0.00","D":"100.00,0.00","M":"48.00,23.50","N":"52.20,24.40","O":"30.00,0.00","Q":"70.00,0.00","P":"50.00,20.00","X":"50.00,22.236","Y":"50.00,-22.236","Z":"50.00,0.00","E":"50.00,24.455","F":"50.00,25.555"}}]},"threeviews":{},"flowChart":{},"function":{}}

1. NLP:

Common stem:[

DiameterRelation{diameter=AC, circle=Circle[⊙O]{center=O, analytic=y\_O=f(x\_O), length=null},

DiameterRelation{diameter=BD, circle=Circle[⊙Q]{center=Q, analytic=y\_Q=f(x\_Q), length=null},

PointRelation:A, PointRelation:B, PointRelation:C,

LineCrossCircleRelation{line=CP, circle=⊙O, crossPoints=[C, M], crossPointNum=2},

LineCrossCircleRelation{line=BP, circle=⊙Q, crossPoints=[B, N], crossPointNum=2}

Sub stem: []

Conclusion:[ProveConclusionRelation:[MultiLineCrossRelation{lines=[DN,AM,XY] }]]]

1. Strategies

Generating 3756 adding auxiliary line strategies based on Strategy Network

|  |  |
| --- | --- |
| 1 | connect point M and point O |
| 2 | connect point N and point Q |
| 3 | create middle point G of segment AM |
| …… | …… |
| 1213 | extended segment DN intersection segment XY at point X\_107 |
| …… | …… |
| 1625 | extended segment AM intersection segment XY at point X\_155 |
| …… | …… |
| 3756 | connect point X\_314 and point X\_352 |

1. Rank strategies by value network

We choose the top 10 candidates as the branching auto solving strategies.

|  |  |
| --- | --- |
| 1 | create middle point G of segment DN, connect point G and point Q |
| 2 | create middle point G of segment AM, connect point G and point O |
| 3 | connect point X and point O |
| 4 | extended segment AM intersection segment XY at point E |
| 5 | extended segment DN intersection segment XY at point F |
| 6 | create vertical segment MG of segment XY through point M which the foot is point G |
| 7 | create vertical segment AG of segment DN through point A which the foot is point G |
| 8 | extended segment AM intersection segment DN at point G |
| 9 | connect point M and point N |
| 10 | connect point N and point Q |

The strategies of number 4 and 5 be validated useful for problem solving.

Human-like solving processes:

1. AutoSolve:[

(1)∵ draw cross point E of AM and XY, draw cross point F of XY and DN

(2)∵ Y, Z, P, X, E, F is collinear

(3)∵ F, N, D is collinear

(4)∵ A, M, E is collinear

(5)∵ BD is the diameter of the circle Q

(6)∵ point N

(7)∴ by(4,5,6): Rt∠BND

(8)∴ by(7): BN⊥DF, pedal point is N

(9)∴ by(8): Rt∠BNF

(10)∵ by(6): △FNP

(11)∴ by(9,10): Rt△FNP(vertex is point N)

(12)∵ ⊙O

(13)∵ ⊙Q

(14)∴ by(12,13): ⊙O cross with ⊙Q

(15)∴ by(14): OQ is the perpendicular bisector of XY

(16)∴ by(15): Rt∠AZP

(17)∵ △BPZ

(18)∴ by(16,17): Rt△BPZ(vertex is point Z)

(19)∵ ∠BPZ and ∠NPX is a pair of vertical angles

(20)∴ by(19): ∠BPZ=∠NPX

(21)∴ by(11,18,20): BP\*NP=FP\*PZ

(22)∵ AC is the diameter of circle O.

(23)∵ point M

(24)∴ by(4,22,23): Rt∠AMC

(25)∴ by(24): AE⊥CM, pedal point is point M

(26)∴ by(25): Rt∠CME

(27)∴ by(23): △EMP

(28)∴ by(26,27): Rt△EMP(vertex is point M)

(29)∴ by(15): Rt∠CZP

(30)∵ △CPZ

(31)∴ by(29,30): Rt△CPZ(vertex is point Z)

(32)∵ ∠CPZ and ∠MPX is a pair of vertical angles

(33)∴ by(32): ∠CPZ=∠MPX

(34)∴ by(28,31,33): MP\*CP=EP\*PZ

(35)∵ points B, X, N, D, Y is concyclic of ⊙Q

(36)∴ by(35): XY is one chord of ⊙Q

(37)∴ by(35): BN is one chord of ⊙Q

(38)∴ by (36,37): (PX)\*(PY)=(BP)\*(NP)

(39)∵ points A, M, X, C, Y is concyclic of ⊙O

(40)∴ by(39): XY is one chord of ⊙O

(41)∴ by(39): CM is one chord of ⊙O

(42)∴ by(40,41): (PX)\*(PY)=(MP)\*(CP)

(43)∴ by(21,34,38,42): point E and point F coincide

(44)∴ by(1,2,3,43): AE, DF, FY intersected at the same point E

]

Example 2 shows the processes of automatically ranking strategies by value network.

**Example 3: An Olympic math problem for algebra**

1. Problem description:

Assume x, y, z are positive numbers and . Prove:

1. NLP:

Common stem: [

AtomAttributeRelation{atomAttribute=AtomAttribute{atomExpr=Express:[x],numberType=POSITIVE}}, AtomAttributeRelation{atomAttribute=AtomAttribute{atomExpr=Express:[y],numberType=POSITIVE}},

AtomAttributeRelation{atomAttribute=AtomAttribute{atomExpr=Express:[z],numberType=POSITIVE}},

InequalityRelation[dualExpressCompare=(x\*y\*z)≥1],

Conclusion:[ProveConclusionRelation:[InequalityRelation[dualExpressCompare=((((x^5)-(x^2)))/(((x^5)+(y^2)+(z^2))))+((((y^5)-(y^2)))/(((x^2)+(y^5)+(z^2))))+((((z^5)-(z^2)))/(((x^2)+(y^2)+(z^5))))≥0]]]]

1. AutoSolve:[

(1)∵P\_e=((((x^5)-(x^2)))/(((x^5)+(y^2)+(z^2))))+((((y^5)-(y^2)))/(((x^2)+(y^5)+(z^2))))+((((z^5)-(z^2)))/(((x^2)+(y^2)+(z^5))))

(2)∵ (x\*y\*z)≥1

(3)∴by(1,2):(((x^2)+(y^2)+(z^2))/(((x^5)+(y^2)+(z^2))))+((((x^2)+(y^2)+(z^2)))/(((x^2)+(y^5)+(z^2))))+((((x^2)+(y^2)+(z^2)))/(((x^2)+(y^2)+(z^5))))≤3

(4)∴by(2,3): (x^5+y^2+z^2)\*(y\*z+y^2+z^2)≥((x^2)\*(x\*y\*z)^(1/2)+y^2+z^2)^2

(5)∴by(4): (x^5+y^2+z^2)\*(y\*z+y^2+z^2)≥(x^2+y^2+z^2)^2

(6)∴by(1,2,3,5): (x^2+y^2+z^2)/(x^5+y^2+z^2)≤(y\*z+y^2+z^2)/(x^2+y^2+z^2)

(7)∴by(1,2,3,5): (x^2+y^2+z^2)/(x^2+y^2+z^5)≤(x\*y+x^2+y^2)/(x^2+y^2+z^2)

(8)∴by(1,2,3,5): (x^2+y^2+z^2)/(y^5+x^2+z^2)≤(z\*x+z^2+x^2)/(x^2+y^2+z^2)

(9)∴by(6,7,8):(x^2+y^2+z^2)/(x^5+y^2+z^2)+(x^2+y^2+z^2)/(x^2+y^2+z^5)+(x^2+y^2+z^2)/(y^5+x^2+z^2)≤2+(x\*y+y\*z+z\*x)/(x^2+y^2+z^2)≤3

(10)∴by(9):((((x^5)-(x^2)))/(((x^5)+(y^2)+(z^2))))+((((y^5)-(y^2)))/(((x^2)+(y^5)+(z^2))))+((((z^5)-(z^2)))/(((x^2)+(y^2)+(z^5))))≥0]

Example 3 shows the human-like solving processes generated by MCTS.